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CAD MODELS FOR MILLIMETER-WAVE SUSPENDED SUBSTRATE MICROSTRIP LINES AND FIN-LINES

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ABSTRACT

Simple and accurate equations for determining the propagation characteristics of suspended and inverted microstrip lines and fin-lines are presented. These equations are vital to the use of computer aided design techniques for such lines and circuits based on these media.

INTRODUCTION

Until recently there have been no simple and accurate models, for computer aided design of planar millimeter-wave circuits using suspended substrate microstrip lines and fin-lines, that cover the entire useful range of operation. Computer aided numerical techniques for microstrips [1] and fin-lines [2] are complicated, time consuming and cumbersome. Therefore, a strong need for the design of such circuits in a lucid and tractable manner has led us to the development of closed-form design equations for insulated [3] and unilateral fin-lines [4]. But till now no such equations were available for suspended microstrip, inverted microstrip and the bilateral fin-line. In the present paper closed-form equations are presented for the propagation constant and characteristic impedance of such lines. The derivations for these equations are not presented here due to lack of space.

SUSPENDED AND INVERTED MICROSTRIP

Analysis

The models of suspended and inverted microstrips [Figs. 1a, 1b] are based on the equation for the impedance of the microstrip in an homogeneous medium Z_0 and equation for the microstrip effective dielectric constant $\varepsilon_e(0)$. Z_0 is given within 0.03% by [5]

$$Z_0 = \frac{\eta_0}{2\pi} \ln \left[\frac{f(u)}{u} + \left(1 + \left(\frac{2}{u}\right)^2\right)^{\frac{1}{2}}\right] \quad (1)$$

$$\text{where } f(u) = 6 + (2\pi-6) \exp\left[-\left(\frac{30.666}{u}\right)^{0.7528}\right] \quad (2)$$

where $u = w/(a+b)$ for suspended microstrip and $u = w/b$ for inverted microstrip, $\eta_0 = 120\pi\ \Omega$.

The characteristic impedance of the microstrip is given by

$$Z = Z_0/\sqrt{\varepsilon_e(0)} \quad (3)$$

For suspended microstrip the effective dielectric constant at zero frequency is obtained from

$$\sqrt{\varepsilon_e}(0) = \left\{1 + \left(a_1 - b_1 \ln\left(\frac{w}{b}\right)\right)\left(\frac{1}{\sqrt{\varepsilon_r}} - 1\right)\right\}^{-1} \quad (4)$$

where

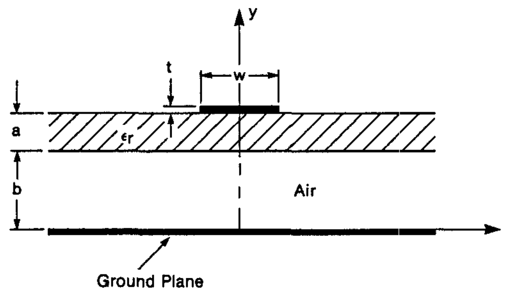

Figure 1a - Suspended Microstrip

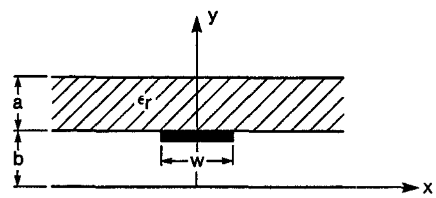

Figure 1b - Inverted Microstrip

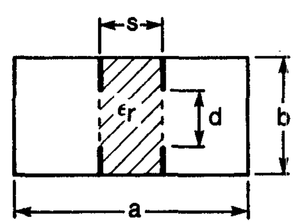

Figure 1c - Bilateral Fin-line

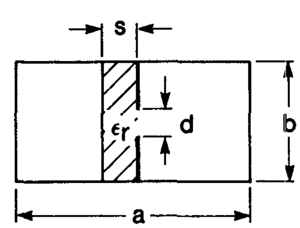

Figure 1d - Unilateral Fin-line

$a_1 = 0.307 + 0.239\ (a/b)$

$\qquad$ for $0.6 \leq a/b \leq 1$

$b_1 = 0.0727 - 0.0136\ (a/b)$

and

$a_1 = 0.155 + 0.505\ (a/b)$

$\qquad$ for $0.2 \leq a/b \leq 0.6$

$b_1 = 0.023 + 0.1863\ (a/b) - 0.194\ (a/b)^2$

For inverted microstrip the effective dielectric constant at zero frqeuency is obtained from

$$\sqrt{\varepsilon_e(0)} = 1 + \{a_1 - b_1 \ln (\tfrac{w}{b})\} (\sqrt{\varepsilon_r} - 1) \qquad (5)$$

where

$a_1 = 0.144 + 0.1231\ (a/b)$

$\qquad$ for $0.6 \leq a/b \leq 1$

$b_1 = 0.0642 + 0.0306\ (a/b)$

and

$a_1 = 0.0669 + 0.2521\ (a/b)$

$\qquad$ for $0.2 \leq a/b \leq 0.6$

$b_1 = 0.02832 + 0.0884\ (a/b)$

The accuracy of these static expressions is better than ± 1% of spectral domain-results [6]. Over the range $1 \leq w/b \leq 8$, $0.2 \leq a/b \leq 1$ and $\varepsilon_r \leq 3.80$, which is the range for most practical applications of these microstrip lines. The inaccuracy in the above equations goes to the order of ± 3% for $\varepsilon_r \approx 10.0$.

<u>Synthesis</u>

If Z, ε_r and a/b are given, then w/b can be obtained for the suspended microstrip by solving the following equation numerically

$$Z = \frac{\eta_0 \{1 + (a_1 - b_1 \ln (\frac{w}{b}))\} (\frac{1}{\sqrt{\varepsilon_r}} - 1)}{(\frac{b}{a+b}) \frac{w}{b} + 1.98 (\frac{b}{a+b})^{0.172} (\frac{w}{b})^{0.172}} \qquad (6)$$

For a desired impedance Z the strip width of the inverted microstrip is given by

$$\frac{w}{b} = \exp \{0.5\ (-p - \sqrt{p^2 - 4q})\} \qquad (7)$$

where

$$p = 0.1446\ b_1 (\sqrt{\varepsilon_r} - 1)\ Z - 8.5581$$

$$q = 18.31 - 0.1446\ (1 + a_1(\sqrt{\varepsilon_r} - 1))Z,$$

$\qquad$ for $1 \leq w/b \leq 3$

and

$$p = 0.1576\ b_1(\sqrt{\varepsilon_r} - 1)\ Z - 8.851$$

$$q = 19.585 - 0.1576\ (1 + a_1(\sqrt{\varepsilon_r} - 1))Z,$$

$\qquad$ for $3 \leq w/b \leq 8$

The synthesis equations are accurate to within ± 1%.

APPLICATION TO ANISOTROPIC SUBSTRATE

It may be shown that the above models are also valid with anisotropic substrates by defining an isotropic equivalence of the anisotropic substrate, where the equivalent substrate thickness $\bar{a}$ and the equivalent permittivity $\bar{\varepsilon}_r$ are given by

$$\bar{a} = a\ [\varepsilon_x / \varepsilon_y]^{\frac{1}{2}} \qquad (8)$$

$$\bar{\varepsilon}_r = [\varepsilon_x / \varepsilon_y]^{\frac{1}{2}} \qquad (9)$$

The method requires that one of the principal axes of the substrate be parallel to the substrate (x-axis) and the other normal to the substrate (y-axis). The strip dimensions remain unchanged. The procedure for utilizing the above models then proceeds through calculation of the mode capacitances, C_m,

$$C_m = 1/(\bar{V}_m\ \bar{Z}_m) \qquad (10)$$

where $\bar{V}_m$ is the mode phase velocity and $\bar{Z}_m$ the mode impedance of the equivalent isotropic substrate. The impedance and effective dielectric constant of the anisotropic microstrip is then computed from

$$Z_a = 1/(C\sqrt{C_m\ C_o}) \qquad (11)$$

$$\varepsilon_e(0) = C_m / C_o \qquad (12)$$

where C_o is the homogeneous mode capacitance of the microstrip and C is the velocity of EM-Wave in the free-space.

FINLINES

<u>Propagation Constant</u>

According to the ridged waveguide model [7] the dominant mode propagation constant in fin-line may be described as

$$\beta = k_o \sqrt{\varepsilon_e(f)} \qquad (13)$$

where $k_o = \frac{2\pi}{\lambda}$ and λ is the free-space wavelength at frequency f,

$$\varepsilon_e(f) = k_e - (\frac{\lambda}{\lambda_{ca}})^2 \qquad (14)$$

k_e is the equivalent dielectric constant at frequency f and λ_{ca} is the cut-off wavelength of the air-filled fin-line of same dimensions. If k_c, the value of k_e at the cut-off frequency, is known then k_e can be computed from the dispersion model of Pramanick and Bhartia [8]. But for low ε_r and thin substrates, k_c can be used for k_e for all practical purposes. k_c is given by

$$k_c \approx (\lambda_{cd}/\lambda_{ca})^2 \tag{15}$$

The normalized cut-off wavelength b/λ_{cd} of the bilateral fin-line is given by (Fig. 1c)

$$\frac{b}{\lambda_{cd}} = \frac{b}{2(a-s)} \left[1 + \bar{N}\,\bar{X}_b\right]^{-\frac{1}{2}} \tag{16}$$

where

$$\bar{N} = (4/\pi)\ \{b/(a-s)\}\ (1 + 0.2\sqrt{\frac{b}{a-s}})$$

$$\bar{X}_b = \ln\csc(\frac{\pi}{2}\frac{d}{b}) + \varepsilon_r\, G_d$$

$$G_d = \eta_d \arctan(\frac{1}{\eta_d}) + \ln[1 + \eta_d^2]^{\frac{1}{2}}$$

$$\eta_d = (s/a)/(b/a)(d/b)$$

The normalized cut-off wavelength $\frac{b}{\lambda_{ca}}$ of the air filled bilateral fin-line is obtained by setting ε_r=1 in equation (16). The accuracy of the above expression is ±0.8% with respect to the spectral domain method [2], for s/a < 1/8.

The normalized cut-off wavelength $\frac{b}{\lambda_{cd}}$ in unilateral fin-line is given by

$$\frac{b}{\lambda_{cd}} = \frac{b}{2(a-s)} \left[1 + \bar{N}\,(1 - \frac{s}{a}\,(-0.0769\,\varepsilon_r + 1.231)\,F(\frac{s}{a}))\ \bar{X}_a\right]^{-\frac{1}{2}} \tag{17}$$

where

$$\bar{X}_a = 2\ln\csc(\frac{\pi}{2}\frac{d}{b}) + \varepsilon_r\,[G_d + G_a]$$

$$G_a = \eta_a \arctan(\frac{1}{\eta_a}) + \ln[1 + \eta_a^2]^{\frac{1}{2}}$$

$$\eta_a = \eta_d\,(d/b)$$

and

$$F(\frac{s}{a}) = -25.1223+31.524[\ln(\frac{a}{s})]-12.504[\ln(\frac{a}{s})]^2 +1.9454[\ln(\frac{a}{s})]^3 \quad \text{for } d/b < 0.5$$

$$= -33.934+42.451[\ln(\frac{a}{s})]-17.057[\ln(\frac{a}{s})]^2 + 2.5885[\ln(\frac{a}{s})]^3 \quad \text{for } 0.5 < d/b < 0.75$$

$$= -48.0487+59.2846[\ln(\frac{a}{s})]-23.77[\ln(\frac{a}{s})]^2 + 3.47[\ln(\frac{a}{s})]^3 \quad \text{for } 0.75 < d/b < 1$$

The cut-off wavelength of the air filled unilateral fin-line is obtained from

$$\frac{b}{\lambda_{ca}} = \frac{b}{2a}\left[1+\frac{4}{\pi}(\frac{b}{a})(1+0.2\sqrt{\frac{b}{a}})\ln\csc(\frac{\pi}{2}\frac{d}{b})\right]^{-\frac{1}{2}} \tag{18}$$

The above equations were compared with spectral domain method [2] and have been found to be accurate to within ±0.8% for s/a < 1/8 and 2.2 < ε_r < 3.8. The propagation constant computed using them have an accuracy of ±2% for all practical purposes.

Characteristic Impedance

According to the ridged waveguide model, fin-line characteristic impedance is given by

$$Z_0 = Z_{0\infty}/\sqrt{\varepsilon_e(f)} \qquad (19)$$

where $Z_{0\infty}$, defined to be the characteristic impedance of the finline at infinite frequency, is a frequency dependent quantity. An expression for $Z_{0\infty}$, using the power-voltage definition of characteristic impedance, for unilateral fin-lines is available in [4]. $Z_{0\infty}$ for small slots $(d/b) \leq 0.2)$ in bilateral fin-line is given, within ±2% of spectral domain results, by

$$Z_{0\infty} = \frac{240\ \pi^2(p\bar{x} + q)\ (b/(a-s))}{(0.385\ \bar{x} + 1.762)^2} \qquad (20)$$

where

$$\bar{X} = \bar{X}_b - G_d(\varepsilon_r - 1),$$

and

$$p = 0.01\ (\frac{b}{\lambda})^2 + 0.097\ (\frac{b}{\lambda}) + 0.04095,$$

$$q = 0.0031\ (\frac{b}{\lambda}) + 0.89.$$

Equation (20) will be useful in matching beam lead devices across the small slot of a bilateral fin-line.

CONCLUSIONS

In the preceding sections accurate closed-form equations have been presented for the analysis of suspended microstrip, inverted microstrip, bilateral fin-lines and unilateral fin-lines. The equations are adequately accurate for all practical purposes and are simple and easy to use on a desk top computer for millimeter-wave circuit designs. Lack of space prevents presentation of the detailed derivations of these equations.

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